SHAPE OF THE BOUNDARY OF A FREELY MOVING PLASMA IN A MAGNETIC FIELD

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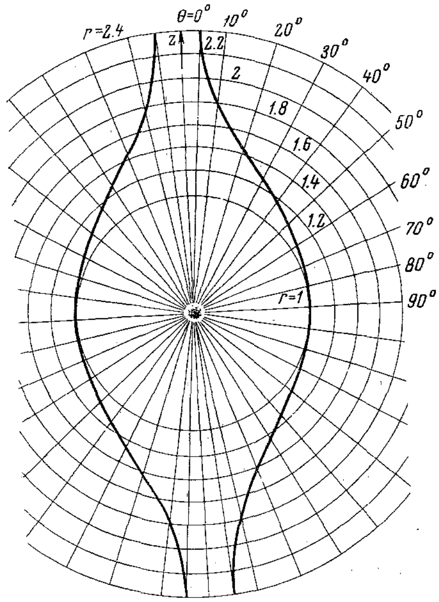
Various processes occur [1] when a plasma expands in a vacuum containing a magnetic field, the discussion relating to a spherical



boundary between the plasma and the field. It is of interest to consider the actual (nonspherical) shape of the boundary. From the condition of pressure equality, we can evaluate the shape of the boundary.

§1. From point O (see figure) a plasma of total mass M expands freely. A mass $M/4\pi d\Omega$ enters a conical solid angle $d\Omega$, and we assume as an approximation that this element is in no way related to the rest of the plasma. The cone has a volume $R^3 d\Omega/3$, in which R is the distance from O to the boundary. The density of the plasma in the cone is taken to be

$$\rho = {}^3/_4\ M / \pi R^3. \qquad (1.1)$$

The external magnetic field B_0 is uniform and is directed along the z-axis before the start of the expansion; it is deformed by the expanding plasma, which displaces it completely. The field exerts a pressure $B^2(\theta)/8\pi$ on the plasma. As $B(\theta)$ we take the value for a field around a superconducting sphere in a uniform external field B_0; at the boundary of the sphere

$$B(\theta) = {}^3/_2\ B_0 \sin\theta. \qquad (1.2)$$

We equate the pressure from the plasma flow to the pressure of the magnetic field:

$$2\rho v^2 \cos^2\psi = {}^1/_8\ B^2(\theta) / \pi, \qquad (1.3)$$

in which $v = (2E/M)^{1/2}$ is the speed of the plasma, E is the energy released at O, and ψ is the angle between the normal to the boundary and the radius R. Differential geometry gives

$$\cos^2\psi = R^2 / (R^2 + R'^2) \qquad (R = R(\theta),\ R' = dR/d\theta). \qquad (1.4)$$

§2. Substitution into (1.3) for ρ, $B(\theta)$, $\cos\psi$, and v of (1.1), (1.2), and (1.4) gives

$$dr/d\theta = -\sqrt{1/\sin^2\theta r - r^2},$$

$$r = R/R_0, \qquad R_0 = ({}^{32}/_3\ E / B_0^2)^{1/3}. \qquad (2.1)$$

This equation has been solved numerically subject to $r(90°) = 1$, which follows from considerations of symmetry. The results are shown in the figure.

§3. Consider the asymptotic behavior of r for $\theta \to 0$; here we neglect the second term under the radicand in (2.1), which gives $r \sim |\ln(\theta/2)|^{2/3}$, i.e., r tends fairly slowly to infinity as $\theta \to 0$.

I am indebted to Yu. P. Raizer for a useful discussion.

REFERENCE

1. Yu. P. Raizer, "Acceleration and energy conversion for a plasma expanding in an empty space containing a magnetic field," PMTF, no. 6, 1963.

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